Criteria equations

Performance criteria are used in several files. Different criterion is given in subass.txt and simass.txt files. In addition criteria can be selected in info.txt. Below is listed the code/heading used in each file together with the equation identificator. Further down all the equations are defined.

Code to equation coupling

Equation IDs for subbasin assessment criteria (subassX.txt):

Heading	Description	Equation ID
NSE	Nash-Sutcliffe efficiency	*NSE*
CC	Pearson correlation coefficient (Kling-Gupta efficiency, part 1)	*CC*
RE(%)	relative bias in percent	*RE%*
RSDE(%)	relative error in standard deviation in percent	*RS%*
Sim	average of simulated variable	*cm*
Rec	average of observed variable	*rm*
SDSim	standard deviation of simulated variable	*cd*
SDRec	standard deviation of observed variable	*rd*
MAE	mean absolute error	*MAE*
RMSE	root mean square error	*RMSE*
Bias	bias	*Bias*
SDE	Error of standard deviation	*ES*
KGE	Kling-Gupta efficiency	*KGE*
KGESD	Kling-Gupta efficiency, part 2	*KGESD*
KGEM	Kling-Gupta efficiency, part 3	*KGEM*
NRMSE	normalised root mean square error	*NE*
NSEW	Nash-Sutcliffe efficiency adjusted for bias	*NSEW*

Equation IDs for simulation assessment criteria (simass.txt):

Name	Code	Equation ID
Regional NSE	RR2	*REGNSE*
Regional RA	RRA	*REGRA*
Regional RE	RRE	*REGRB*
Regional MAE	-	*REGMAE*
Average NSE	MR2	*AVNSE*
Average RA	MRA	*AVRA*
Average RE	MRE	*AVRB*
Average RSDE	MRS	*AVRSB*
Average CC	MCC	*AVCC*
Average ARE	MAR	*AVARB*
Spatial NSE	SR2	*SPATNSE*
Spatial RA	RRA	*SPATRA*

Name	Code	Equation ID
Spatial RE	-	*SPATRB*
Kendalls Tau	TAU	*AVTAU*
Median NSE	MD2	*MEDNSE*
Median RA	MDA	*MEDRA*
Median KGE	MKG	*MEDKGE*
Median NRMSE	MNR	*MEDNE*
Mean NSEW	MNW	*AVNSEW*

Equation IDs for calibration simulation assessment criteria (bestsims.txt and allsim.txt):

Heading	Description	Equation ID
rr2	regional Nash-Sutcliffe efficiency (data from all subbasins combined in one data series)	*REGNSE*
sr2	spatial Nash-Sutcliffe efficiency, calculated using annual means for all subbasins (requires at least 5 years and 5 subbasins with data) to form one data series to calculate the Nash-Sutcliffe efficiency on	*SPATNSE*
mr2	average of Nash-Sutcliffe efficiencies for subbasins	*AVNSE*
rmae	regional mean absolute error (data from all subbasins combined in one data series)	*REGMAE*
sre	spatial relative bias (calculated on annual means for all subbasins)	*SPATRB*
rre	regional relative bias (data from all subbasins combined in one data series)	*REGRB*
mre	average of the relative bias for all subbasins (Note: fraction, not %)	*AVRB*
rra	regional RA, similar to regional NSE, RA is a Nash-Sutcliffe like criterion where the square in the Nash-Sutcliffe formula is exchanged with a coefficient value	*REGRA*
sra	spatial RA, similar to spatial NSE, RA is a Nash-Sutcliffe like criterion where the square in the Nash-Sutcliffe formula is exchanged for a coefficient value	*SPATRA*
mra	average value of RA for subbasins, RA is a Nash-Sutcliffe like criterion where the square in the Nash-Sutcliffe formula is exchanged with a coefficient value	*AVRA*
tau	average of Kendall's Tau value for subbasins	*AVTAU*
md2	median of Nash-Sutcliffe efficiency for subbasins	*MEDNSE*
mda	median of all subbasins' RA (Nash-Sutcliffe like criteria where the square is exchanged with a coefficient value)	*MEDRA*
mrs	average of error in standard deviation for subbasins	*AVRSB*
mcc	Pearson correlation coefficient, average of all subbasins with observations	*AVCC*
mdkg	median of Kling-Gupta efficiency (MKG in info.txt) for subbasins	*MEDKGE*
mare	average of absolute relative bias for subbasins (Note: fraction. not %) (MAR in info.txt)	*AVARB*
mnr	median of normalised RMSE for subbasins	*MEDNE*
mnw	average of Nash-Sutcliffe efficiencies adjusted for bias for subbasins	*AVNSEW*

Equation IDs for performance criteria set in info.txt are tabled here.

Equation definitions

Denotations

c	computed value
r	recorded value
i	index for time steps with observations in a time series of a station
mi	number of values in a time series of a station
j	index of stations
mj	number of stations
ij	index over time steps with observations for all stations
mij	number of time steps with obsevations for all stations
cm	average value of $c_i, i=1, mi$ for a station
rm	average value of $r_i, i=1, mi$ for a station
cd	standard deviation of $c_i, i=1, mi$ for a station
rd	standard deviation of $r_i, i=1, mi$ for a station

Basic equations

Average value for a time series of a station:

$$xm = \frac{1}{mi} \sum_{i=1}^{mi} x_i$$

x=*r* or *c*

Standard deviation of a time series of a station:

$$xd = \sqrt{\frac{1}{mi} \sum_{i=1}^{mi} x_i^2 - xm^2}$$

x=*r* or *c*

Criteria equations for a time series of a station

Nash-Sutcliffe Efficiency (*NSE* or R2):

$$NSE = 1 - \frac{\sum_{i=1}^{mi} \left(c_i - r_i \right)^2}{\sum_{i=1}^{mi} \left(r_i - rm \right)^2}$$

Efficiency with coefficient a (*RA*):

$$RA = 1 - \frac{\sum_{i=1}^{mi} \left| c_i - r_i \right|^a}{\sum_{i=1}^{mi} \left| r_i - rm \right|^a}$$

Bias:

$$Bias = \frac{\sum_{i=1}^{mi} \left(c_i - r_i \right)}{mi}$$

Relative bias (*RB* or RE):

$$RB = \frac{\sum_{i=1}^{mi} \left(c_i - r_i \right)}{\left| \sum_{i=1}^{mi} r_i \right|}$$

Relative bias in percent (*RE%*):

$$RE\% = RB \times 100 = \frac{\sum_{i=1}^{mi} \left(c_i - r_i \right)}{\left| \sum_{i=1}^{mi} r_i \right|} \times 100$$

Error of standard deviation (*ES*):

$$ES = cd - rd$$

Relative error of standard deviation (*RS*):

$$RS = \frac{cd - rd}{rd}$$

Relative error of standard deviation in percent (*RS%*):

$$RS\% = RS \times 100 = \frac{cd - rd}{rd} \times 100$$

Mean absolute error (*MAE*):

$$MAE = \frac{\sum_{i=1}^{mi} \left| c_i - r_i \right|}{mi}$$

Kling-Gupta efficiency (*KGE*):

$$KGE = 1 - \sqrt{\left(CC - 1\right)^2 + \left(\frac{cd}{rd} - 1\right)^2 + \left(\frac{cm}{rm} - 1\right)^2}$$

Pearson correlation coefficient, Kling-Gupta efficiency part 1 (*CC*):

$$CC = \frac{\frac{1}{mi}\sum_{i=1}^{mi}\left(r_i \times c_i\right) - cm \times rm}{cd \times rd}$$

Kling-Gupta efficiency part 2 (*KGESD*):

$$KGESD = \frac{cd}{rd}$$

Kling-Gupta efficiency part 3 (*KGEM*):

$$KGEM = \frac{cm}{rm}$$

Root mean square error (*RMSE*):

$$RMSE = \sqrt{\frac{1}{mi}\sum_{i=1}^{mi}\left(c_i - r_i\right)^2}$$

Normalised root mean square error (*NE*):

$$NE = \frac{\sqrt{\frac{1}{mi}\sum_{i=1}^{mi}\left(c_i - r_i\right)^2}}{\max\left(r_i\right)}$$

Kendalls rank correlation coefficient, tau-b, with adjustments for ties (*TAU*):

$$TAU = \frac{n_c - n_d}{\sqrt{\left(n_0 - n_1\right)\left(n_0 - n_2\right)}}$$

Nash-Sutcliffe Efficiency adjusted for bias (*NSEW*). Introduced in Lindström (2016):

$$NSEW = NSE - \frac{Bias^2}{rd^2}$$

where

n_c = number of concordant pairs ($c_i < c_k \, and \, r_i < r_k \, or \, c_i > c_k \, and \, r_i > r_k, i = 1, mi \, k = 1, mi$)

n_d = number of discordant pairs ($c_i < c_k \, and \, r_i > r_k \, or \, c_i > c_k \, and \, r_i < r_k, i = 1, mi \, k = 1, mi$)

n_0 = number of compared pairs

n_1 = number of compared pairs that ties in the computed values

n_2 = number of compared pairs that ties in the recorded values

Criteria equations for a model domain (several stations)

Average Nash-Sutcliffe efficiency (*AVNSE*):

$$AVNSE = \frac{1}{mj} \sum_{j=1}^{mj} NSE_j$$

Median Nash-Sutcliffe efficiency (*MEDNSE*):

$$MEDNSE = median \, j = 1 mj \, NSE_j$$

Spatial Nash-Sutcliffe efficiency (*SPATNSE*):

$$SPATNSE = 1 - \frac{\sum_{j=1}^{mj} \left(cm_j - rm_j \right)^2}{\sum_{j=1}^{mj} \left(rm_j - \frac{1}{mj} \sum_{j=1}^{mj} rm_j \right)^2}$$

Regional Nash-Sutcliffe efficiency (*REGNSE*):

$$REGNSE = 1 - \frac{\sum_{ij=1}^{mij} \left(c_{ij} - r_{ij} \right)^2}{\sum_{ij=1}^{mij} \left(r_{ij} - \frac{1}{mij} \sum_{ij=1}^{mij} r_{ij} \right)^2}$$

Average efficiency with coefficient a (*AVRA*):

$$AVRA = \frac{1}{mj} \sum_{j=1}^{mj} RA_j$$

Median efficiency with coefficient a (*MEDRA*):

$$MEDRA = median\, j=1 mj RA_j$$

Spatial efficiency with coefficient a (*SPATRA*):

$$SPATRA = 1 - \frac{\sum_{j=1}^{mj} \left| cm_j - rm_j \right|^a}{\sum_{j=1}^{mj} \left| rm_j - \frac{1}{mj} \sum_{j=1}^{mj} rm_j \right|^a}$$

Regional efficiency with coefficient a (*REGRA*):

$$REGRA = 1 - \frac{\sum_{ij=1}^{mij} \left| c_{ij} - r_{ij} \right|^a}{\sum_{ij=1}^{mij} \left| r_{ij} - \frac{1}{mij} \sum_{ij=1}^{mij} r_{ij} \right|^a}$$

Average relative bias (*AVRB*):

$$AVRB = \frac{1}{mj} \sum_{j=1}^{mj} RB_j$$

Regional relative bias (*REGRB*):

$$REGRB = \frac{\sum_{ij=1}^{mij} \left(c_{ij} - r_{ij} \right)}{\left| \sum_{ij=1}^{mij} r_{ij} \right|}$$

Spatial relative bias (*SPATRB*):

$$SPATRB = \frac{\sum_{j=1}^{mj} \left(cm_{j} - rm_{j} \right)}{\left| \sum_{j=1}^{mj} rm_{j} \right|}$$

Median Kling-Gupta efficiency (*MEDKGE*):

$$MEDKGE = median\, j = 1 mj KGE_{j}$$

Median of Normalised root mean square error (*MEDNE*):

$$MEDNE = median\, j = 1 mj NE_{j}$$

Average of absolute relative bias (*AVARB*):

$$AVARB = \frac{1}{mj} \sum_{j=1}^{mj} \left| RB_{j} \right|$$

Average Pearson correlation coefficient (*AVCC*):

$$AVCC = \frac{1}{mj} \sum_{j=1}^{mj} CC_{j}$$

Average relative error of standard deviation (*AVRSB*):

$$AVRSB = \frac{1}{mj} \sum_{j=1}^{mj} RS_{j}$$

Average Kendalls rank correlation coefficient (*AVTAU*):

$$AVTAU = \frac{1}{mj} \sum_{j=1}^{mj} TAU_{j}$$

Regional mean absolute error (*REGMAE*):

$$REGMAE = \frac{\sum_{ij=1}^{mij} \left| c_{ij} - r_{ij} \right|}{mij}$$

Average Nash-Sutcliffe efficiency adjusted for bias (*AVNSEW*):

$$AVNSEW = \frac{1}{mj} \sum_{j=1}^{mj} NSEW_j$$

References

Lindström, G., 2016. Lake water levels for calibration of the S-HYPE model. Hydrology Research 47.4:672-682. doi: 10.2166/nh.2016.019.